

# The Inverse Relationship Between Productivity and Activity: A Mathematical Definition of Work Efficiency

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## Abstract

We formalize the relationship between *productivity*, *activity*, and *work* as distinct but interdependent variables in human and organizational performance systems. Defining productivity as the ratio of work output to total activity,  $P = \frac{W}{A}$ , yields the immediate result that productivity and activity are inversely correlated when work is held constant. This framework demonstrates that high activity does not imply high productivity, and that truly efficient systems minimize activity while maximizing work accomplished per unit effort. The conclusion is both mathematical and cultural: it is better to be *productive* than to be *busy*.

## 1 Definitions

- **Activity** ( $A$ ) — the total measurable motion, effort, or time expended.
- **Work** ( $W$ ) — the net transformation or value created; the useful output.
- **Productivity** ( $P$ ) — the efficiency ratio between work accomplished and activity performed:

$$P = \frac{W}{A}.$$

## 2 Inverse Correlation

Assuming  $W$  is approximately constant within a given context, we have:

$$\frac{dP}{dA} = -\frac{W}{A^2} < 0.$$

Hence, productivity decreases as activity increases. The relationship is strictly inverse: the more motion without corresponding work, the less efficient the system.

### 3 Interpretation

Productivity is not additive but fractional. Activity alone has no intrinsic value; it acquires meaning only through the ratio by which it produces work. Excessive meetings, emails, and task-switching raise  $A$  without necessarily increasing  $W$ , thereby lowering  $P$ . Mathematically:

$$A \uparrow \Rightarrow P \downarrow \quad (\text{if } W \text{ constant}).$$

### 4 Optimal Work Zone

For a given system, productivity peaks when the marginal gain in work equals the marginal cost of activity:

$$\frac{dW}{dA} = \frac{W}{A}.$$

At this equilibrium, every unit of activity contributes equally to work. Beyond this point, additional activity produces diminishing or negative returns.

### 5 Corollaries

1. If  $A = 0$ , no work is performed; productivity is undefined (division by zero).
2. If  $W = 0$  but  $A > 0$ , productivity is zero: pure busyness.
3. If  $W > 0$  and  $A$  approaches its minimum viable level,  $P \rightarrow \infty$ : pure efficiency.

### 6 Cultural Implication

The widespread valorization of activity—*being busy*—mistakes motion for progress. In truth, activity is the *denominator* of productivity: the smaller it becomes for a given quantum of work, the greater the outcome. Hence, organizations and individuals obsessed with visible effort often underperform those optimized for invisible efficiency.

### 7 Conclusion

*Productivity is not about how much we do, but how little it takes to achieve real work.* The mathematics is simple:

$$P = \frac{W}{A}.$$

The wisdom is ancient: anyone can be active; only the wise are productive.